Love & Math

An introduction to romantic optimization

Man is now seen to be an enigma only as an individual. In the mass he is a mathematical problem.

-Robert Chambers

All you need is love. All you need is love. All you need is love. Love is all you need.

-The Beatles

Problem 1

Being too sexy

In 1623, Johannes Kepler had a problem.

Kepler

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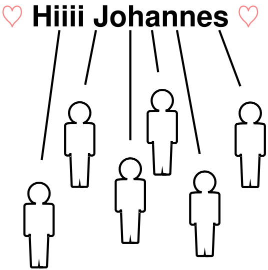
Kepler



He was too sexy.

Rules for dating

- There are a fixed number of suitors.
- He can go on a date with a suitor, after which he will know how they compare to previous suitors.
- After each date, he has to either **marry** or **move on**.
  - If he marries, he can't go on more dates.
  - If he moves on, he can't go back.

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Nothing but the best for Johannes

How can Kepler maximize his chance to marry the **best suitor**?

After any given date, either the suitor is the **best so far** or not.

If they aren't the best so far...

They definitely aren't the best. Keep looking!

If they are the best so far...

There is a k/n chance they are the best overall, if it's the kth date and there are n suitors.

So...how many dates should Kepler go on before settling down?

The optimal strategy [Bruss, 1984]

Given n suitors, the optimal strategy is the following.

- Automatically move on from the first $\lceil n/e \rceil$ dates ($\sim 37\%$).
- Marry the next suitor who is the best so far.

The probability of marrying the best is at least $1/e \sim 37\%$.

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Historical footnote

Sadly, Kepler's dilemma predates this solution by 361 years, and Euler's introduction of his constant by 111 years. Instead, Kepler went on 11 dates and then begged suitor #5 to take him back. They were very happy together.

Problem 2

Making everyone happy(ish)

Why are we being so selfish? Let's try to make everyone happy!

The dating pool

- There are equal populations of men and women.
- Everyone is attracted to the opposite gender, and they can rank their preferences.

How can everyone get married as happily as possible?

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Disclaimer

Everyone being attracted to the opposite gender is necessary for our specific solution to work, not a moral judgement.

...as happily as possible?

Not everyone can get their first choice. What if everyone has a crush on Andrew?

A lesser goal is for everyone to be in a **stable marriage**.

Stable marriages

No two people would rather be married to **each other** than to **their current partners**.

So, even if Alice prefers Andrew to her husband Bob, it's **stable** as long as Andrew doesn't prefer Alice to his wife.

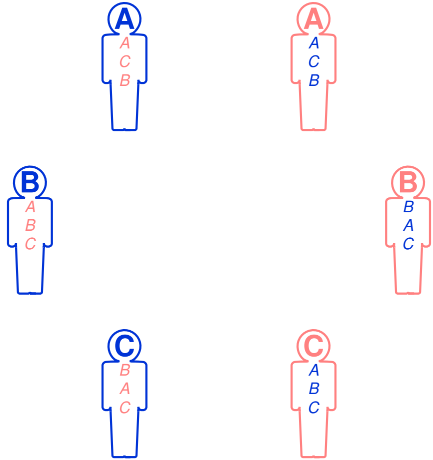
The stable marriage algorithm (Gale-Sharpley, 1962)

The algorithm takes place over rounds, in which **engagements** get made and broken. During each round:

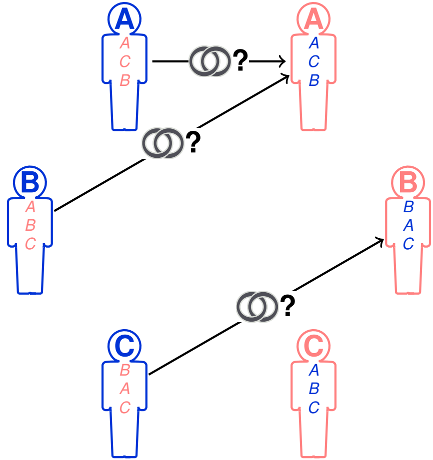
- Each unengaged man **proposes** to the woman they like best who they haven't yet proposed to.
- Each woman **accepts** the proposal from man she likes best, possibly breaking her current engagement.

When everyone is engaged, have a **big group wedding**.

Our dating pool and their preferences



Round 1 proposals



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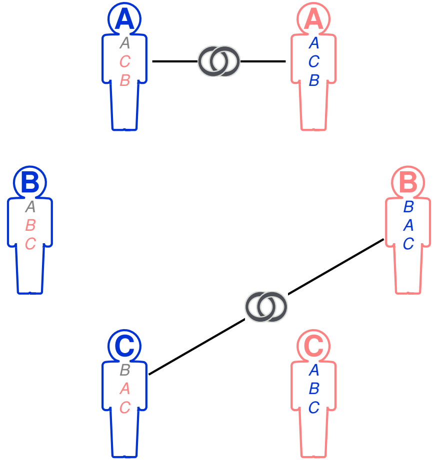
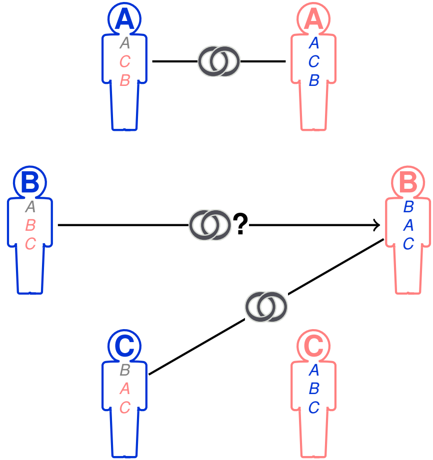


Round 2 proposals



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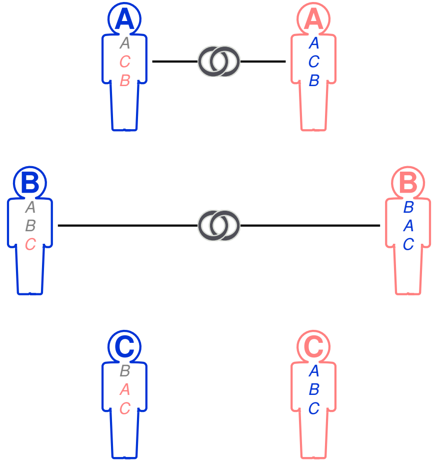
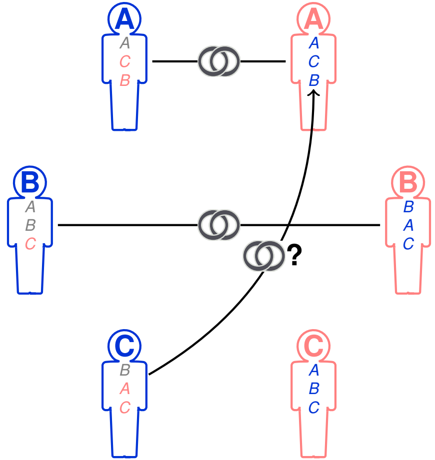


Round 3 proposals



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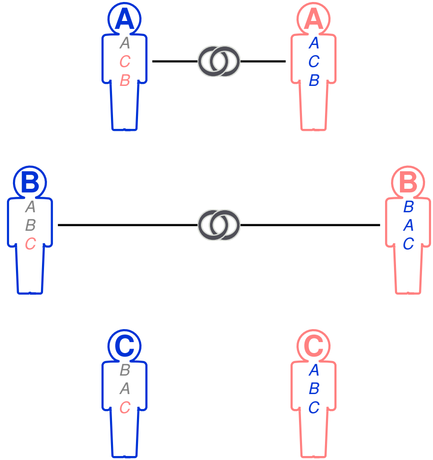
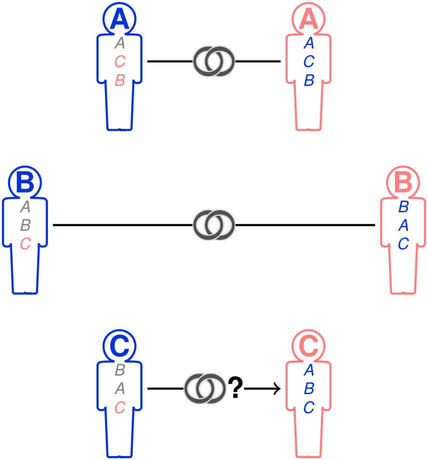


Round 4 proposals



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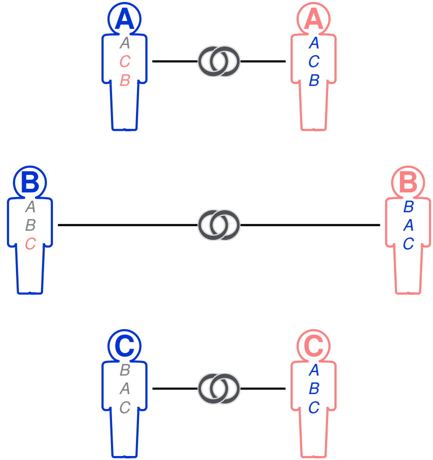
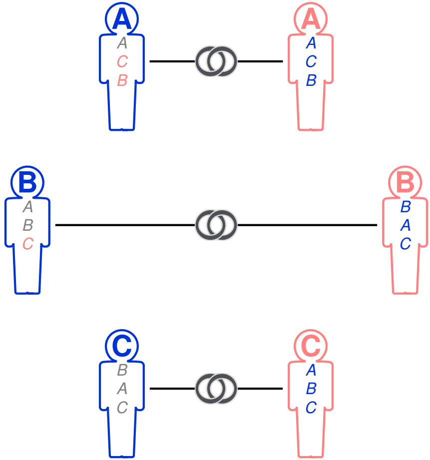


Happy(ish) ever after!



Widely applicable beyond romance

The **stable marriage algorithm** is widely used in school admissions, roommate assignments, content delivery networks, and many other resource allotment problems.

Asymmetry

There can be many stable marriage configurations. This algorithm produces the one most favorable to the **proposers**.

What a coincidence that it resembles the system we have now.

Problem 3

Sharing a dessert

Two paramours are sharing a delicious cake, when they stumble upon an apparent paradox.

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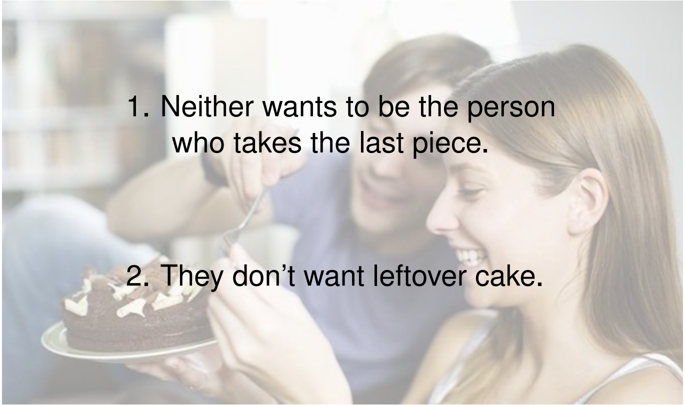

100% practical solution

Take turns eating half of whats left twice as fast.

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- The first person eats half the cake in, say, 60 seconds.
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- The first person eats half of what's left in 15 seconds.

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- And so on.

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Total cake eaten:

$$50\% + 25\% + 12.5\% + ... = \textbf{100\%}$$

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Total time in which to eat it:

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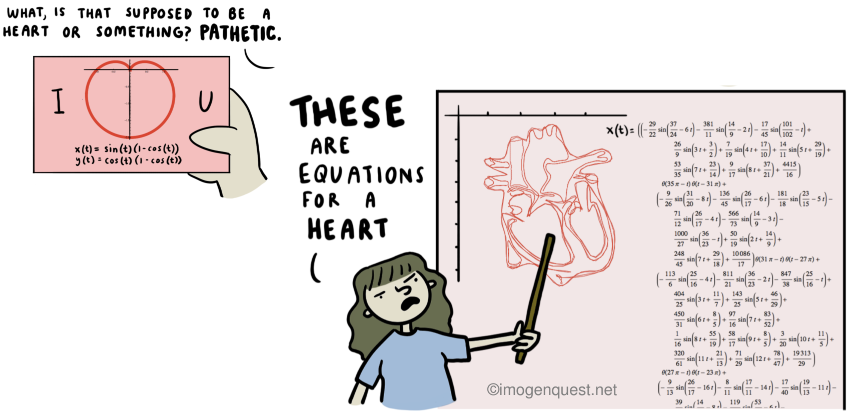
Last bite: **No one.**

♡ Thank you ♡



WHAT, IS THAT SUPPOSED TO BE A HEART OR SOMETHING? PATHETIC.
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U
x(t) = sin(t)(1-cos(t))
y(t) = cos(t)(1-cos(t))
THESE ARE EQUATIONS FOR A HEART
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